The author introduced polar coordinates (ρ, θ) of a point in the elliptical bonded region and determined the stress intensity factor along the polar radius through point (A) in the form

$$k_1 = \frac{P^*}{2\pi(ab)^{1/2}}(a^2\cos^2\theta + b^2\sin^2\theta)^{-1/4}. \tag{3}$$

The apparent similarity between eqns (1) and (3) has led the author to the conclusion that eqn (1) is incorrect! The symbol ϕ should not be confused with the polar angle θ. In fact, the angles ϕ and θ are related as follows:

$$\begin{aligned} a\cos\phi &= c(\theta)\cos\theta \\ b\sin\phi &= c(\theta)\sin\theta \end{aligned} \tag{4}$$

where $c(\theta) = ab/(a^2\sin^2\theta + b^2\cos^2\theta)^{1/2}$ stands for the distance OA. He further casts eqn (3) along the normal to the crack border and arrives at

$$k_1 = \frac{P^*}{2\pi(ab)^{1/2}}\left[\frac{a^2\sin^2\theta + b^2\cos^2\theta}{a^4\sin^2\theta + b^4\cos^2\theta}\right]^{1/4} \tag{5}$$

to which he claims as a new formula for the stress intensity factor. Inserting eqns (4) into eqn (5) leads to eqn (1). This completes the proof for the stress intensity factor. Hence, there is nothing wrong with eqn (1) as the author was simply confused with the angles ϕ and θ. His conclusion has no foundation and should be dismissed.

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AUTHOR'S CLOSURE

Mathematics is a powerful tool: it gives us the ability to make almost any formula correct. Here is how. Suppose we have a formula

$$k = F(\theta)$$

where θ is the angle between the axis Ox and the polar radius. It is easy to make another formula

$$k = f(\phi)$$

also correct. It suffices to assume that $\phi = f^{-1}F(\theta)$. This is precisely what Kassir and Sih did in their rebuttal. There is though one drawback: ϕ can no longer be interpreted as the angle between the polar radius and the axis Ox. If Kassir and Sih wish to claim that their formulae are correct, then they have to admit that all the figures in their book, depicting an ellipse, and all the statements in the book, indicating ϕ as the angle between the polar radius and the axis Ox, are incorrect (see pp. 74, 75, 94, 95, 96, 107, 118 of Kassir and Sih (1975)). It is difficult to believe that two scientists of such stature do not realize that there is a total contradiction between their description of ϕ and Fig. 1 of the rebuttal (which are both correct), and similar statements and figures in the book.

Here is a good question to ask. Suppose, there is a need for a second edition of the book. Either all the formulae or all the drawings and related statements need to be changed. What would the authors do? If one decides that the angle between the polar radius and the axis Ox is a more rational choice than the parameter ϕ, which does not have such clear geometrical interpretation, then the only decision is to change formulae thus admitting that they are incorrect indeed.

Conclusion: if one insists that the argument in the formulae should be interpreted as the angle between the polar radius and the axis Ox, then my results are correct, and the formulae of Kassir and Sih are incorrect, otherwise one can write an infinite number of different formulae, each of them being correct.

It is worth mentioning that my paper was finished in March 1986, and immediately sent to both Kassir and Sih for their comments. Neither responded. One may just wonder why it took them so long to respond. Another paper of mine, in which some other errors of Kassir and Sih were found, has now been submitted for publication. This paper was sent to them both in August 1987. Both are invited to respond. Neither have so far. Kassir and Sih have published an excellent book, and as any book containing a lot of new material, it is bound to have some errors and misprints, and it has. The authors should be grateful to, rather than angry with, someone who undertakes the job of verification.

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